Effects of Parametric Uncertainty on Airfoil Limit Cycle Oscillation

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Introduction

THE need for revolutionizing methods of assessing aeroelastic stability has become increasingly pressing in recent years. This is driven primarily by two factors: 1) the desire to reduce the total cost of certification by reducing testing requirements, and 2) the emergence of unique design concepts to provide impressive performance in military applications. A common feature of these designs is that they substantially increase the potential for nonlinear behavior beyond levels that can be adequately addressed by current engineering tools and processes.

These needs and concerns were the focus of a recent workshop organized by the Air Force Office of Scientific Research and the Air Force Research Laboratory.‡ The workshop addressed traditional areas of concern, such as the basic physics and computational requirements of nonlinear aeroelasticity, but it also included sessions on model verification and validation (V&V) and the role of uncertainty quantification (UQ) in understanding the physics of nonlinear aeroelasticity and certifying aeroelastic stability. The participants of the workshop developed a strong consensus that UQ must play a prominent role in the future of aeroelasticity research; notably, it was agreed that UQ could provide a common language for promoting communication between analysts and test personnel.

This Note is intended to demonstrate the application of standard probability concepts and Monte Carlo simulation (MCS) to the study of airfoil limit cycle oscillation (LCO), which results from a subcritical Hopf bifurcation induced by including a nonlinear spring in the pitch degree of freedom (DOF). Unsteady aerodynamic forces are represented by the R. T. Jones approximation[1] of the circulatory lift. This simple aeroelastic model permits an assessment of aeroelastic performance sensitivity and variability within the context of a well-understood system; furthermore, because each MCS realization requires time integration, the use of a simple model is computationally expedient. Employing such an elementary model is justified in this application because our primary goals are to illustrate the qualitative consequences of uncertainty in a nonlinear aeroelastic system and also to provide a simple example of how probabilistic aeroelastic analyses might be performed in the future. The authors and a colleague have employed essentially the same procedure detailed herein to highlight the influence of various uncertainties in panel LCO.[2–4]

Problem Formulation

The aeroelastic system studied here is a nonlinear incarnation of the standard symmetric airfoil with pitch, $\alpha(t)$, and plunge, $h(t)$, DOF (see Fig. 1). The deterministic structural and aerodynamic model is an extension of that employed by Lee et al.[5] in which the plunge DOF has linear stiffness but the pitch DOF includes a third-order stiffness term in addition to the linear component. For their model, the restoring moment associated with the torsional spring is expressed as $K_\alpha(\alpha + k_{\alpha_3}\alpha^3)$, where K_α is a dimensional stiffness constant and k_{α_3} is the dimensionless parameter governing the third-order term. The reader should note that positive values of k_{α_3} yield a supercritical Hopf bifurcation for which LCOs are sustained at values of u larger than critical (a value u^* determined from linear analysis), whereas negative values of k_{α_3} represent a situation of spring softening for which the bifurcation is subcritical and LCO states occur below u^*.

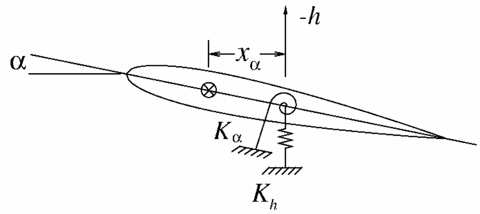

Fig. 1 Two DOF airfoil.

The model employed in the current study includes a fifth-order pitch stiffness term k_{α_5} leading to a restoring moment of the form $K_\alpha(\alpha + k_{\alpha_3}\alpha^3 + k_{\alpha_5}\alpha^5)$. With this form of the torsional stiffness, selection of $k_{\alpha_3} < 0$ and $k_{\alpha_5} > 0$ yields a subcritical bifurcation that possesses a cyclic fold at u^{LCO} such that LCO states are available for $u \geq u^{\mathrm{LCO}}$ where $u^{\mathrm{LCO}} < u^*$. The potential for subcritical bifurcation is of particular interest owing to the associated abrupt increases in LCO amplitude.

The dimensionless equations of motion can be written as a system of first-order differential equations,[5]

$$\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t} = \boldsymbol{F}\big(\boldsymbol{X}; u, k_{\alpha_3}, k_{\alpha_5}\big) \tag{1}$$

where $\boldsymbol{X}$ is an array of eight variables (four variables governing pitch and plunge and four additional variables arising from modeling of the aerodynamics) and u is the dimensionless reduced velocity parameter. This parameter is defined by $u \equiv U/b\omega_\alpha$, where U is the dimensional flight speed, b is the airfoil semichord, and ω_α is the natural pitch frequency (defined by the linear torsional spring stiffness and the elastic axis moment of inertia). The time variable is made dimensionless using U and b as velocity and length scales, respectively.

Equation (1) is efficiently integrated in time using a standard, explicit, first-order time-accurate scheme with fixed time step of 0.002. This choice is observed to yield solutions that are converged in time step and exhibit LCOs in about 1 s of computational time on a conventional desktop computer. LCO amplitudes are obtained by extracting peak pitch angles from computed time histories over the last 10% of the total integration time (2000 time units).

Parametric uncertainty was modeled in the third- and fifth-order stiffness coefficients, k_{α_3} and k_{α_5}, of the pitch spring. No formal attempt was made to base the structural coefficients or their assumed variability on a known physical system. The parameters were assumed to be Gaussian random variables with mean values of -3.0 and 20.0, respectively; each was assumed to have a coefficient of variation of 0.10. Uncertain initial conditions were also studied through randomness in the initial angle-of-attack, $\alpha_0 = \alpha(t = 0)$, which was assumed to be Gaussian with a mean of 0.0 rad and a standard deviation of 0.2 rad. For this system, α also represents the elastic deflection in pitch, so that α_0 can be interpreted as a measure of the initial strain energy.

Each of the random parameters was assumed to be independent. This assumption is reasonable for α_0, but it is unclear whether this assumption is justified for k_{α_3} and k_{α_5}. Intuition suggests that these parameters should be correlated, but the nature of this correlation and its consequences are difficult to anticipate. The authors plan to examine this issue in the future. Standard MCS was employed to generate realizations of the airfoil's response over time. Results presented in the next section are for 4000 realizations at five reduced velocities from 5.75 to 6.50.

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Results

Figure 2 is a "random" bifurcation diagram, which summarizes the MCS results. This figure shows the estimated LCO amplitude probability density function (PDF) for several reduced velocities. Because no attempt has been made to base the structural coefficients or their assumed variability on a physical system, only qualitative conclusions are drawn from this figure, which is intended to illustrate an informative presentation of the MCS results for a nonlinear aeroelastic system.

Among these results, foremost is the ability to estimate the probability that LCO will exceed a specified threshold at a given reduced velocity. Information presented in this form could someday be the basis for a risk-based flutter design criterion, but in current design and certification frameworks it is useful primarily as a vivid indicator of risk. For example, deterministic analysis of the baseline system predicts zero steady-state response (i.e., a stable focus) for $u = 5.75$, which is approximately 8% below the linear instability at $u = 6.26$ and also is below the lower limit point on the unstable LCO branch ($u = 5.95$); however, MCS predicted a small but nonzero probability of encountering LCO at this reduced velocity. The LCO probability is even higher at reduced velocities of 5.90 and 6.00, which are still well below the baseline bifurcation point. The risk induced by the parametric uncertainties is reflected clearly in the bimodal shape of the amplitude PDF at these reduced velocities.

Figure 3 is a scatterplot of the random system parameters that illustrates the separation of the realizations into classes based on whether or not LCO was observed. In doing so, it clearly displays the regions of the input parameter space that should be avoided to prevent LCO. It is apparent that the occurrence of LCO in this system depends strongly on α_0 and that this dependence is essentially symmetric with respect to zero pitch. The reader should note that a standard deterministic analysis would correspond to only one point on this diagram. Even if a particular combination of system parameters and initial conditions does not exhibit LCO (e.g., any of the blue points in Fig. 3), it is clear that the relative risk of the design cannot be definitively assessed without the context provided by the clusters of points in the scatterplot.

Figures 4a and 4b display the same points as Fig. 3, but projected on the (α_0, k_{α_3}) and (α_0, k_{α_3}) planes, respectively. It is observed that for increasingly negative values of k_{α_3}, the α_0 required to induce LCO becomes smaller in absolute value. Also, it is clear that k_{α_5} has little influence on the occurrence of LCO. Results not illustrated here show that the LCO amplitude decreases almost linearly with increasing k_{α_5}. These observations agree completely with intuition, which suggests that the negative third-order stiffness permits the subcritical bifurcation and the positive fifth-order stiffness restrains

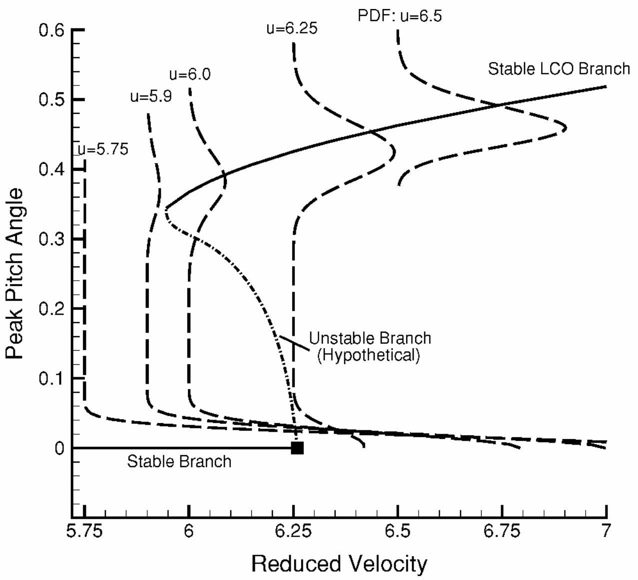

Fig. 2 Response of pitch-and-plunge airfoil. The solid curve is LCO amplitude of baseline airfoil. Dashed curves show estimated probability density functions from Monte Carlo simulation at each dynamic pressure.

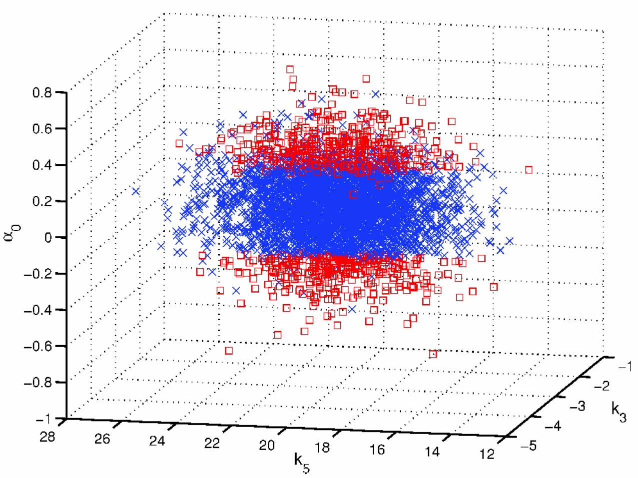

Fig. 3 Scatterplot of the k_{α_3}, k_{α_5}, and α_0 samples separated into LCO and non-LCO classes. The red squares correspond to realizations that resulted in LCO; the blue crosses indicate realizations that converged in time to equilibrium at zero deflection.

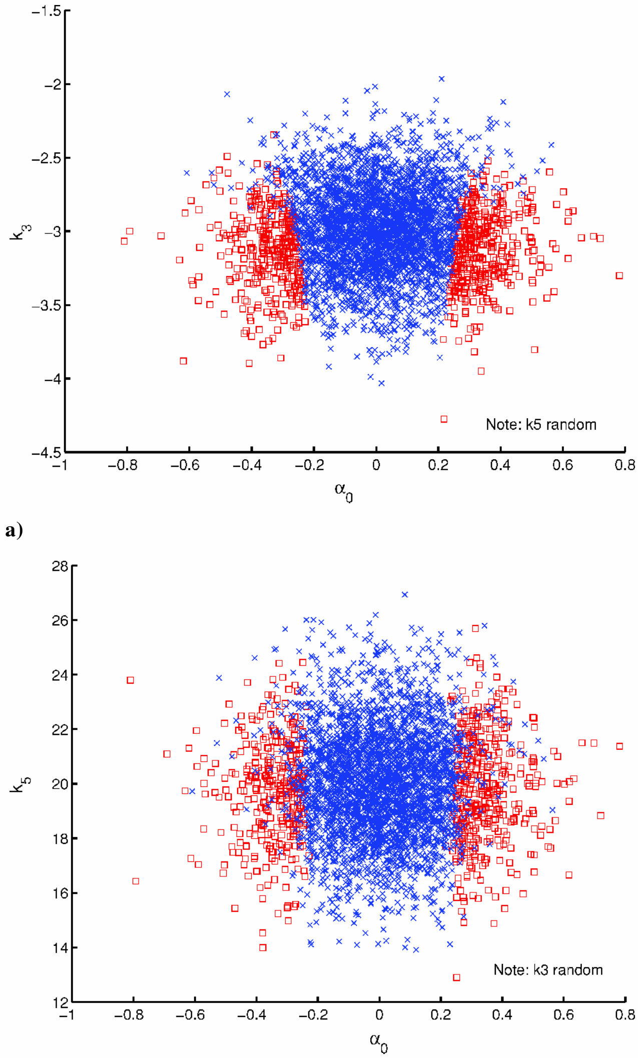

Fig. 4 Projections of Fig. 3 on the (α_0, k_{α_3}) and (α_0, k_{α_5}) planes, with samples separated again into LCO and non-LCO classes: a) α_0 and k_{α_3}, and b) α_0 and k_{α_5}.

the resulting LCO, much as a third-order term would in a supercritical bifurcation.

Conclusions

The reader should notice that the analysis and results were obtained and presented in an intuitive manner that could be extended readily to the study of more complex systems. MCS clearly permits an existing model to be used for uncertainty analysis with minimal modifications. The stochastic bifurcation diagram is simply an extension of standard bifurcation diagrams. The assessment of how particular input uncertainties are reflected in the response uncertainty was performed entirely as a postprocessing step.

Key concerns that ought to be addressed in a thorough study of any complex system include identifying uncertain inputs and accurately quantifying their variability, developing an analytical representation of the system that sufficiently captures the relevant physics, establishing adequate V&V procedures and metrics for the computational implementation of the model, and selecting postprocessing methods that maximize insight while simultaneously avoiding the destruction of information that might not be immediately recognizable.

The relative importance and difficulty of addressing each of these concerns depend both on the problem being studied and on the intended uses of the results. Practical considerations typically dictate that sufficient information cannot be obtained for a complete representation of uncertainty in the input variables. Developing a dependable and thorough V&V procedure for computational mechanics in the presence of uncertainty is an area of current research and substantial debate over terminology and philosophy.

References 2–4 and other recently presented research have shown that relatively minor levels of variability in system parameters, loads, and boundary conditions can induce significant changes in the stability of nonlinear aeroelastic systems. Consequently, because uncertainty quantification provides a consistent basis for assessing system performance and technological risks, it should be a cornerstone of aeroelasticians' efforts to stimulate fundamental advances in airframe design.

Acknowledgments

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